Are eVTOL Aircraft inherently more susceptible to the Vortex Ring State than Conventional Helicopters?

R.E. Brown

There is currently extensive interest worldwide in developing small, lightweight, electrically-powered, multi-rotor vertical takeoff and landing aircraft. The aim is to use these new vehicles to carry a small number of passengers on short-range intra-urban missions, for instance as part of a passenger ferry service between an airport and a downtown commuter hub. The concern is that these aircraft might have certain characteristic design features that, in combination with the environmental conditions that they will experience, will render them particularly susceptible to a potentially hazardous operating regime, known as the Vortex Ring State, especially during their descent and landing. This paper extends our classical understanding of the basic physics that underpins the Vortex Ring State in order to assess the likely impact of this phenomenon on the safety and operational characteristics of this new class of vehicle.

ARE eVTOL AIRCRAFT INHERENTLY MORE SUSCEPTIBLE TO THE VORTEX RING STATE THAN CONVENTIONAL HELICOPTERS?

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Principal notation

A	:	rotor disc area, πR^2
a	:	speed of sound
$C_{L_{max}}$	:	maximum blade sectional lift coefficient
c	:	mean blade chord
c_d	:	disturbance growth rate
g	:	acceleration due to gravity
k	:	parameter in Perry's VRS model
M_{tip}	:	Mach number at the blade tip
m	:	aircraft mass
N	:	number of rotor blades
Q, q	:	VRS margin
q_x, q_z	:	VRS gust margins
$\bar{q}_x$, $\bar{q}_z$	:	VRS gust margins, scaled by v_h
R	:	rotor radius
T	:	rotor thrust
t	:	time
$\boldsymbol{v}_d$	:	disturbance convection rate
v_i	:	rotor induced velocity
v_h	:	estimate for the hover induced velocity given by momentum theory, *i.e.* $\sqrt{W_\rho}$
v_x	:	aircraft horizontal speed, or rotor edgewise velocity
v_z	:	aircraft vertical speed (negative in descent), or rotor axial velocity
W_ρ	:	altitude-weighted disc loading, $(T/A)/(2\rho)$
$\boldsymbol{x}_d$	:	location of disturbance centre
$\boldsymbol{x}_R$	:	location of rotor
$\bar{\lambda}_i$	:	v_i, scaled by v_h
$\bar{\mu}_x$, $\bar{\mu}_z$	:	v_x, v_z scaled by v_h
$\bar{\mu}_W$	:	parameter in Perry's VRS model
ρ	:	air density
σ	:	rotor solidity, *i.e.* Nc/A

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1 INTRODUCTION

The Vortex Ring State (VRS) is a potentially hazardous regime of operation that a rotorcraft can enter during the critical descent and landing phases of flight, as well as potentially in manoeuvres at low forward speeds. Practically, the Vortex Ring State is encountered when the translational velocity up through one or more of the rotors of the aircraft becomes comparable in strength to the velocity that is induced by the rotor itself. The rotor wake can then be induced to collapse into an unsteady, disordered vortical structure rather than persisting in the well-ordered, helical form that is usually imagined. The wake, once disrupted in this fashion, interacts with the rotor in an essentially chaotic way that causes not only a loss of thrust on the rotor but also significant fluctuations in the forces and moments that are produced by the system. It seems straightforward to imagine that these changes in the loads on the rotor will influence the dynamics of the aircraft, but perhaps not so obvious is the fact that the strong coupling between the aerodynamics of the wake and the loads that are produced on the system can lead to counterintuitive aircraft behaviour in some cases, particularly as the control system or pilot tries to counteract or escape from the phenomenon. For this reason there is an extensive history of helicopter accidents that are related to the onset of the Vortex Ring State and to the subsequent inability of the pilot to escape its effects.

1.1 Helicopter Experience

An early realisation[1] was that the extent of the region of the flight envelope that might be affected by the VRS, as defined on a plot of the forward speed against the descent rate of the aircraft, scales directly with the (square root) of the disc loading of the rotorcraft (where the disc loading is defined

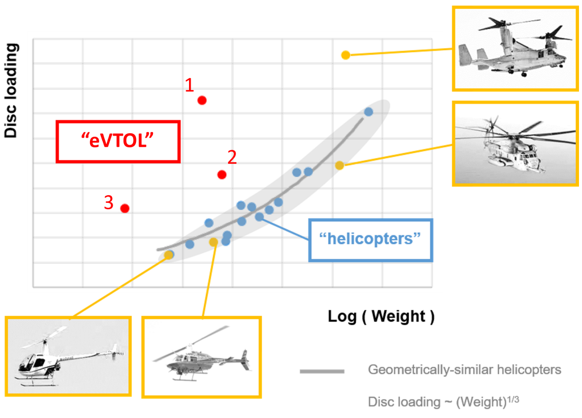


Figure 1
Correlation between disc loading and aircraft weight for conventional helicopters and eVTOL designs.

as the vehicle's weight divided by the total area of its lifting rotors). A simple scaling argument shows also that the disc loading of conventional helicopters should increase in proportion to the cube root of their weight. This relationship is shown in Fig. 1 and reveals the close correlation of the range of well-known conventional helicopters with the expected trend. Combining these two scaling laws helps to explain the commonly-held belief that, for small, light rotorcraft, the region of the flight envelope that is affected by VRS is relatively small, and perhaps might even be confined to forward speeds and descent rates that are too low to be of practical concern. In this view, the VRS becomes a problem only when the operating characteristics of larger aircraft, for example military transports and oil-rig ferries are considered. Actual accident statistics do not support this view, yet possibly *because* the causes, symptoms and effects of the VRS remain largely under-appreciated (and indeed a subject of confusion) within many parts of the helicopter community, the implications of the Vortex Ring State for the safety of *all* rotor-borne flight seem also not to have percolated as far as they should have within the eVTOL community. Indeed it is likely that a new generation of designers may hardly be aware of the important ramifications of the onset of this flow condition for the operational safety of the various new vehicles that are currently on the drawing boards of many companies, large and small, around the world.

1.2 Why are eVTOL aircraft different?

Although accurate and reliable values for the weight and disc area of actual eVTOL aircraft tend to be a closely guarded commercial secret, it is possible to infer some data from information that is in the public domain. The disc loading and weight for several different eVTOL designs for which reliable data could be established are plotted in Fig. 1 alongside the data for more conventional rotorcraft. Examination of the plot suggests, especially if this small sample of eVTOL aircraft can be taken as representative, that this new class of vehicle might be considerably different in their characteristics when compared to the conventional rotorcraft for which we might claim to have an extensive understanding of their aerodynamic qualities and foibles. This new class of vehicles seems invariably to have relatively low weight (see Fig. 1), but also, compared to conventional rotorcraft, a much higher disc loading. This feature of eVTOL designs can be understood as a natural consequence of certain design decisions, for instance the requirement to balance the lifting capability of the rotors when in hover against their efficiency as propellers in forward flight.

This simple graph suggests that the designers of eVTOL aircraft, particularly those that follow this trend towards low weight but high disc loading, might do to consider carefully the significance of the associated expansion in the region of the flight envelope of their aircraft that might be affected by the VRS. This paper attempts to map out some of the consequences of this expansion on the performance and safety of these vehicles when in descending flight.

An early lesson with rotorcraft that did not obey the simple cube-root scaling law between weight and disc loading was given by the V-22 Osprey tilt-rotor. For several good reasons, this aircraft was designed with a significantly higher disc loading for its weight than might be suggested by the design trend for conventional helicopters shown in Fig. 1. The unfavourable characteristics of the vehicle in descent were exposed in an accident in April 2000. This accident led to an intensive programme of remedial work and theoretical investigation[2], which, not incidentally, continues to underpin much of our understanding of the practical consequences of the VRS.

Unfortunately, some learnings from the V-22 experience remain under-exploited. Preliminary work along a range of avenues suggested that several specific rotorcraft design features might be highly suspect in exacerbating some of the characteristics of the Vortex Ring State that might be encountered by any particular vehicle. High blade twist was implicated in creating a more severe VRS[3], and certain tip shapes are known to alter the rate at which the instabilities within the rotor wake, that are now known to be responsible for the onset of the VRS, spread and strengthen[3]. The side-by-side rotor configuration is known to be particularly problematic, especially in terms of the effects of roll rate in inducing premature VRS onset, and, if the aircraft enters the VRS asymmetrically as a result, in precipitating troublesome interactions between the air data and

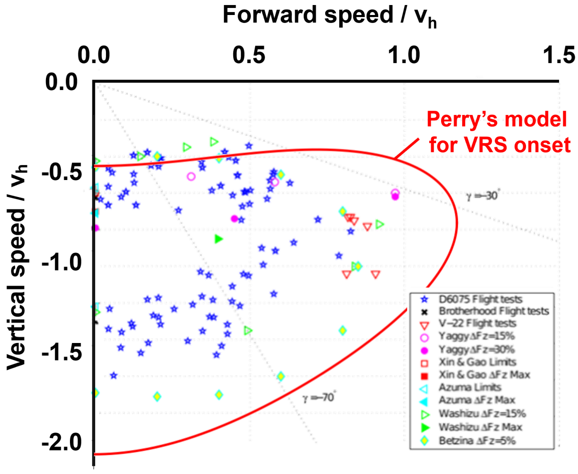


Figure 2
Perry's model for the VRS onset boundary, showing good agreement with available data.

control systems of the vehicle - particularly if they are not designed to be resilient to such effects. Additionally, the aerodynamic interactions between the wakes of closely-spaced rotors is thought to lead to coupling within the system whereby loss of stability in the wake of one rotor can cascade through the system to cause premature collapse of the wakes of other nearby rotors.

The fact that the rotors of eVTOL aircraft are naturally driven towards propeller-like twist distributions for reasons of efficiency in forward flight, and to the incorporation of tip shapes that are unconventional (at least by helicopter standards) to minimise acoustic signature or to achieve the perceived benefits of large tip vortex separation in maximising the figure of merit in hover, should motivate the designers of these vehicles to be fully cognizant of the likely effects of these design decisions on the likely performance of their vehicles in descending flight. Furthermore, most of these aircraft are extremely compact and often exploit the Distributed Electric Propulsion concept in one form or another. This invariably results in a design that has multiple, small rotors, often spread out along the length of the wing but nearly always in close proximity to each other and to the other components of the aircraft. This aircraft configuration inherently embodies those design features that are thought to be particularly pernicious in terms of exacerbating the aircraft's susceptibility to the VRS, and any designer of such a vehicle would do well to quantify the likely effects of the VRS on their creation by performing the appropriate analysis.

Finally, operation deep within the urban landscape will expose these new rotorcraft to a highly unsteady aerodynamic environment. The aircraft will inevitably be subject to large and rapid shifts in wind direction and speed, and the issue will be particularly acute when operating close to large buildings and structures. Classical helicopter experience shows that sudden tailwinds and updraughts can shift the rotors very suddenly from their normal operating state into a flow regime that, within short order, will precipitate the onset of the VRS. For this reason, the introduction of a metric that allows the robustness of this class of vehicle to VRS-induced upset to be quantified might go a long way to proving and certifying the airworthiness of a specific aircraft design, especially if matched to the known environmental conditions near the end-points of its intended route.

This paper will make the first steps in that direction by defining a measure for VRS susceptibility that is somewhat analogous to the gust envelope that is used within the fixed-wing aircraft community to assess the resilience of the aircraft to atmospheric perturbations.

2 THEORETICAL UNDERSTANDING

Several theoretical models exist that attempt to locate the boundary for VRS onset on the flight envelope of the rotor when the state of the system is described in terms of the aircraft's forward speed and climb rate[4]. Brown *et al.*[5] postulated a model (here called Perry's model given this co-author's fundamental contribution to its derivation), formulated largely on heuristic grounds, that related the forward speed and descent rate of the helicopter to a critical rate of transport within the flow at which the rotor wake would collapse from its usual orderly state to precipitate the onset of the VRS. According to this model, a rotor will enter the VRS once it crosses a lobe-shaped boundary on the flight envelope defined in terms of the climb rate v_z and forward speed v_x of the system.

2.1 Analytical Model for VRS Onset

In mathematical terms, Perry's boundary for VRS onset can be expressed as the locus of points $(\bar{\mu}_x, \bar{\mu}_z) = (v_x/v_h, v_z/v_h)$ on a non-dimensional version of the flight envelope (see Fig. 2) that satisfy the equation

$$(k\bar{\mu}_x)^2 + (\bar{\mu}_z + \bar{\lambda}_i)^2 = \bar{\mu}_W^2 \qquad (1)$$

A crucial point to realise is that the known dependence of the size of the VRS domain on the disc loading is represented explicitly within this model, at least to first order, through the non-dimensionalisation of the axes of the flight envelope in terms of the estimate of the hover induced velocity v_h that is given by simple momentum theory.[1]

[1]This follows since, from simple momentum theory, $T = 2\rho A v_h^2$ or, on rearranging, $v_h = \sqrt{W_\rho}$ where $W_\rho = (T/A)/(2\rho)$ is the (altitude-weighted) disc loading of the rotor.

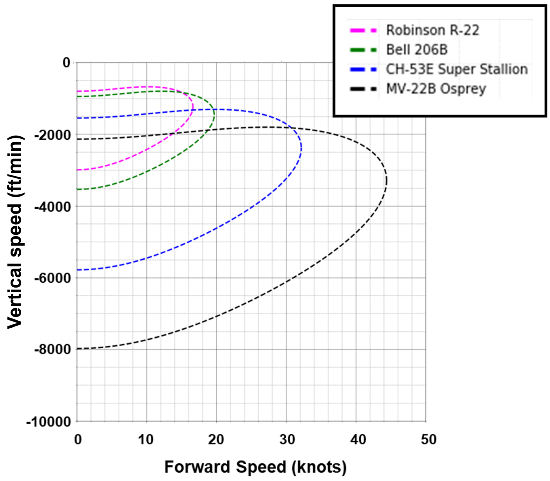


Figure 3
Predicted VRS onset boundaries for various well-known conventional helicopters.

Equation 1 thus defines a VRS boundary that is universal to the extent that the same two constants $\bar{\mu}_W$ and k define the onset of the VRS independently of the detailed design characteristics (such as the twist distribution and tip shape of its blades) of the particular rotor that is being considered. The model's characterisation of the onset of the VRS is thus inherently a little blunt, but if values of 0.76 and 0.65 are adopted respectively for $\bar{\mu}_W$ and k, then very satisfactory correlation is obtained with a broad cross-section of the various model and full-scale measurements that have been gathered over the years to map out the extent of the VRS regime within the descending flight envelope of conventional helicopters (Fig. 2). In practice, Perry's model has thus provided a powerful engineering-level tool that can be used very rapidly to assess the dangers posed by the onset of the VRS to a given rotorcraft under specified operational conditions.

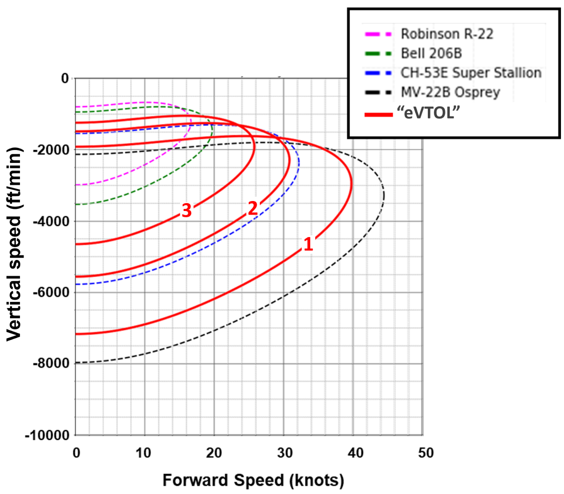


Figure 4
Predicted VRS onset boundaries for several eVTOL aircraft.

Since the model's introduction in the early '00s, some important further theoretical advances have been made. Of most relevance to the subject of this paper is Ahlin and Brown's[6] addition of rigour to the analysis by expressing the empirical constants in Perry's model in terms of the rate of convection and growth of the products of the natural instability of the rotor wake, and their subsequent extension of the analysis to accelerated flight conditions. These two authors also conducted detailed numerical experiments that verified the hypothesis underlying the model[7] and explored some of its practical consequences for helicopter operations. These advances are exploited later in this paper, but for the moment we rely on the predictions of the original model to provide a preliminary, global assessment of the susceptibility of helicopters and eVTOL aircraft to the VRS.

2.2 Analytic results

Figure 3 shows the extent of the flight envelope that is affected by the VRS, as predicted by Perry's model for several well-known conventional rotorcraft. The data shows the effect of disc loading on the size of the region of the flight envelope in which the vehicle might encounter the VRS. According to the model, the large, heavy CH-53E military transport helicopter might experience the VRS at descent rates between 1500 and 6000 ft/min, when flying at speeds less than 30 knots, whereas a small rotorcraft, such as the Bell Jet Ranger, would only encounter the VRS at descent rates between 500 and 1500 ft/min, when flying at speeds less than 20 knots. Given that the disc loading of the V-22 is much higher than that of the other machines that are represented in the figure, the region of the flight envelope that is affected by the VRS is proportionally larger.

Figure 4 shows the same data as in Fig. 3 but with the VRS-affected regions for the three eVTOL aircraft represented in Fig. 1 overlain for comparison with the characteristics of the more conventional rotorcraft. The figure shows very clearly the effect of the very high disc loading of these vehicles in extending the region of the flight envelope that might be affected by the VRS out to flight speeds and descent rates that are comparable with the critical speeds for the largest transport helicopters (and even the V-22 in the most extreme case).

For this reason, it is very unlikely that experience gained in VRS avoidance with equivalent-sized conventional helicopters will be applicable in the eVTOL domain. It is also highly unlikely though that a fundamental reappraisal of the physical effects at play, perhaps extending back even to a re-analysis of the effects of wake instability on the susceptibility of this class of vehicle to the VRS, will be necessary. Indeed, it is much more likely that much of what we do already know will be transferable into the new domain once re-interpreted in the correct light. This paper attempts to make a start along those lines.

3 THE AIRCRAFT'S TRAJECTORY

It is important to realise from the outset that the enlargement of the VRS domain for systems with high disc loading is not in itself inherently 'bad'. Although the minimum forward speed at which the VRS will be encountered increases in proportion to the (square root of the) disc loading, so too does the margin between level flight and the descent rate at which the VRS will first be encountered when flying at low forward speed. An ideal situation could be imagined thus where a suitably-designed aircraft could exploit this effect, and would simply require careful management of its trajectory throughout the descent and landing manoeuvre as it negotiates the deepened, but also elongated, 'safe' corridor on the flight envelope that is endowed by its higher disc loading.

Although current design practice for eVTOL aircraft does appear to be to have disc loadings that are significantly higher than those for conventional helicopters, a careful analysis suggests that the disc loadings of the current generation of eVTOL aircraft might not in fact be *high* enough for the vehicles to be able to exploit effectively the associated broadening of their VRS corridor[2]. Indeed it is rather more likely, for the reasons set forth below, that the VRS characteristics of eVTOL aircraft as currently designed will in fact be exacerbated by their higher disc loading.

3.1 Direct effects of disc loading

The direct effects of disc loading on the aircraft's susceptibility to the VRS can be illustrated in practical terms by defining a representative descent trajectory and exploring the effects of disc loading on how close the associated velocities of the aircraft take it to the boundary for VRS onset. For present illustrative purposes we model the vertical and horizontal velocities along the trajectory as

$$\begin{aligned} v_z &= -\frac{H}{T}\sin^2(\pi(t/T)^{m+1})/H_* \\ &= -\frac{H}{T}\,V_z^{(m)}(t/T) \end{aligned} \tag{2}$$

and

$$\begin{aligned} v_x &= \frac{L}{T}\,(1-(t/T))^n/L_* \\ &= \frac{L}{T}\,V_x^{(n)}(t/T). \end{aligned} \tag{3}$$

[2]The engineering estimate $W_\rho = a^2(\sigma/6)C_{L_{max}}M_{tip}^2$ for the altitude-weighted disc loading allows the limits on practical design to be explored. The reader is encouraged to substitute sensible values into this simple formula to determine for themselves the latitude for further practical increases in this parameter.

In these expressions, H, L and T are respectively the height through which the aircraft descends, the distance covered during the descent and the time taken to descend. Scaling factors $H_* = \int_0^1 \sin^2(\pi\tau^{m+1})d\tau$ and $L_* = 1/(n+1)$ are introduced to maintain consistency within the kinematics. The two parameters $-\infty < m < \infty$ and $0 \le n < \infty$ define the shape of the trajectory. The highest descent rates are biased towards the beginning of the descent if $m < 1$ and towards the end of the descent if $m > 1$. Similarly, the larger the value of n, the more the horizontal deceleration of the aircraft is biased towards the end of the descent.

The effect of disc loading on the susceptibility of the system to the VRS along its trajectory is most neatly expressed by re-scaling Eq. 2 and Eq. 3 onto the non-dimensionalised flight envelope that has axes $(\bar{\mu}_x, \bar{\mu}_z) = (v_x/v_h, v_z/v_h)$. Additional insight is also obtained by re-casting the mathematical description of the trajectory in terms of the glide-slope angle and non-dimensional aircraft velocity rather than expressing it more directly in terms of the Cartesian components of the aircraft's velocity.

The tangent of the glide-slope angle of the aircraft along its trajectory, as mapped onto the non-dimensionalised flight envelope, is given by

$$\begin{aligned} \tan\bar{\gamma} &= -\bar{\mu}_z/\bar{\mu}_x \\ &= -v_z/v_x \\ &= \frac{H}{L}\,\Gamma^{(m,n)}(\tau) \end{aligned} \tag{4}$$

where $\Gamma^{(m,n)} = V_z^{(m)}/V_x^{(n)}$ and $0 \le \tau \le 1$. The glide slope angle γ on the 'real' flight envelope (v_x, v_z) is thus the same at any point along the aircraft's trajectory as it is at the equivalent point along the trajectory as mapped onto the non-dimensional flight envelope $(\bar{\mu}_x, \bar{\mu}_z)$.

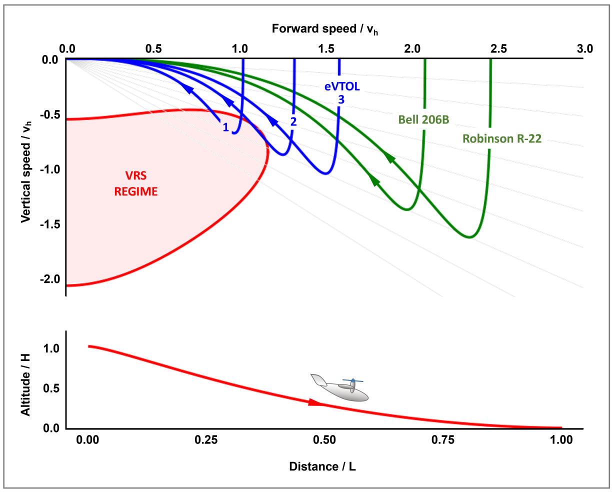
The speed of the aircraft along its trajectory, again as mapped onto the non-dimensional flight envelope, is given by Pythagoras's rule as

$$\begin{aligned} \bar{\mu}^2 &= \bar{\mu}_x^2 + \bar{\mu}_z^2 \\ &= (v_x/v_h)^2 + (v_z/v_h)^2 \\ &= \frac{1}{W_\rho}\,[v_x^2 + v_z^2] \\ &= \frac{1}{W_\rho}[(\frac{L}{T})^2 W_x^{(n)}(\tau) + (\frac{H}{T})^2 W_z^{(m)}(\tau)] \end{aligned} \tag{5}$$

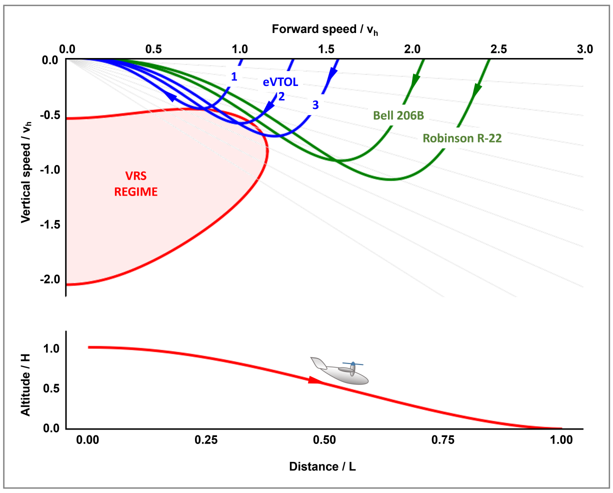
where $W_x^{(n)} = (V_x^{(n)})^2$ and $W_z^{(m)} = (V_z^{(m)})^2$ and on exploiting the definition of v_h in terms of the altitude-weighted disc loading W_ρ of the system.

Figure 5 shows representative behaviour of the system for three different example descent trajectories (generated using different values of m and n - the

(a) “shallow descent”



(b) “aggressive descent”



(c) “steep descent”

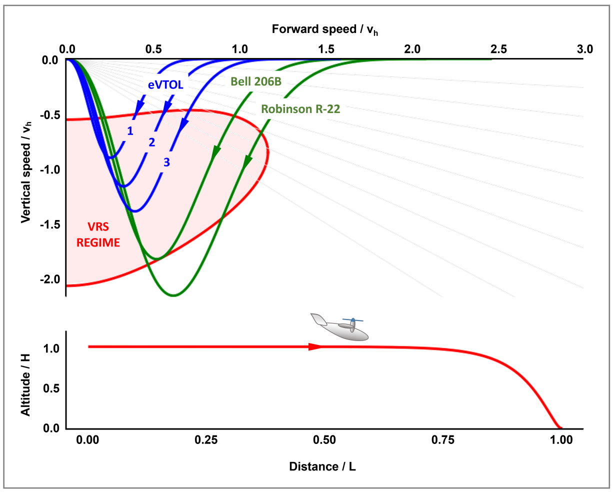


Figure 5
Descent trajectories for several helicopters and eVTOL aircraft mapped onto Perry’s VRS onset boundary.

non-dimensional representation can of course be converted into real numbers by substituting suitable values of L, H, T and W_ρ).

The value of the non-dimensional representation of the flight envelope is readily apparent: as the disc loading of the system is varied, the VRS boundary as delineated on the non-dimensional flight envelope $(\bar{\mu}_x, \bar{\mu}_z)$ remains static whereas the trajectory of the aircraft is simply expanded or shrunk radially (about the origin $(0, 0)$ of the flight envelope) in proportion to the parameter $1/W_\rho$. The fundamental shape of the trajectory is otherwise left unchanged by this scaling.

The interaction of the aircraft with the VRS regime is rather different in each of the three cases shown. It is important to bear in mind that once the aircraft trajectory crosses the VRS onset boundary, the aerodynamics of the vehicle changes character markedly and becomes highly nonlinear. In most cases it is very unlikely, in fact, that, once having entered the VRS domain, the vehicle will continue to follow the simple dynamics that is implied by the trajectories shown in the figures.

Bearing this important caveat in mind, Fig. 5(a) shows a shallow descent trajectory where a small conventional helicopter of the size of most current eVTOL concepts would find itself with a substantial margin against the onset of the VRS. It is reasonably clear in this case how a design with somewhat increased disc loading would fare in terms of a reduction of its margin of safety against the onset of the VRS. Figure 5(b) shows a more aggressive descent where a conventional small helicopter would find itself very close to entering the VRS over part of its trajectory - and where an aircraft with comparatively higher disc loading would in all likelihood have to take corrective action to address the onset of VRS on its rotors.

Figure 5(c) shows a trajectory which terminates in a steep final descent to landing. This trajectory would prove extremely dangerous for any conventional helicopter, precipitating it into VRS under conditions that might prove very difficult to escape from without significant loss of altitude. It is interesting to note however that many current plans for the exploitation of the eVTOL concept focus on limiting environmental impact, particularly as far as noise is concerned. There is thus strong motivation, given this worthy aspiration, to adopt a steep final trajectory, both to limit the aircraft’s acoustic footprint on the ground as well as potentially to mitigate the BVI noise that is generated by rotors when operating at shallow descent angles. In addition, some of the current business models for this class of vehicle will require the aircraft to descend steeply in order to access a network of highly-constrained landing sites that are buried deep within the urban landscape. Results such as those presented here may come as no surprise to the operators of conventional rotorcraft, but should suggest to the promoters of some eVTOL concepts that their aspirations for minimum environmental impact and commercial utility may need to be tempered by the need for the trajectory of the aircraft to remain well clear of the boundary for VRS onset.

Regulators in particular may need to bear in mind that experience gained in helicopter operation may not transfer readily to the eVTOL domain in this respect. Instead, a fundamentally different approach might need to be taken to designing the trajectories that these new aircraft will follow during their descent and landing so that the particular characteristics of this class of aircraft with respect to their susceptibility to the VRS along the intended flightpath are explicitly taken into account. Some tools that may be useful in this respect are set out in Section 4 of this work.

Note that this first-pass analysis is most directly applicable to true helicopters and to those vehicles that have so-called 'multicopter' format - in other words those vehicles where all the rotors remain relatively horizontal throughout the flight trajectory. Although many prospective eVTOL aircraft (at least at the time of writing this paper) still fit fairly well into this characterisation, this configuration was perhaps somewhat more typical of the first generation of eVTOL aircraft. Indeed, a somewhat more sophisticated approach may be needed to expose some of the VRS-related characteristics of the vehicles that have evolved from this earlier class of machines.

3.2 Effect of Manoeuvres

A useful first embellishment to the conceptualisation of the physics of these vehicles is to realise that VRS onset is a local phenomenon, dependent on the speed of the airflow that is encountered by the rotors themselves - rather than, as has been assumed implicitly in the analysis presented so far, on the flow velocity that is experienced by the vehicle as a whole. To accommodate this observation, and to be able to analyse the characteristics of more modern eVTOL aircraft, particularly those that embody concepts such as distributed propulsion and multi-rotor or hybrid wing-rotor aerodynamics where the individual rotors may not all be subject to the same aerodynamic environment or be controlled in the same way, a more detailed rendition of the dynamics of the vehicle is undoubtedly necessary.

In the following section of the paper, two examples are given where resolving the dynamics of the rotors independently of the dynamics of the vehicle as a whole gives interesting additional insights into how the particular design characteristics of some of these vehicles might render them particularly susceptible to the onset of the VRS during their descent and landing.

The first example presented here considers the fundamental VRS-related characteristics of the commonly encountered 'vectored thrust' eVTOL configuration. When in high speed forward flight, this type of vehicle is lifted by a set of wings and the rotors are tilted forward into 'propeller' mode to provide the forward thrust that is required to sustain the vehicle in flight. During the deceleration of the vehicle that accompanies its descent and landing, a point is reached where the wings can no longer generate lift efficiently and so the thrust axis of the rotors is rotated upwards to produce a component of vertical force to supplement the contribution from the lifting surfaces. In this 'helicopter' mode, deceleration of the vehicle is achieved principally through the drag that is created by the lifting surfaces, as supplemented by effective rearwards tilt of (some of) the rotors. This rearwards tilt is provided either through the vehicle being flared nose-up or through further rearwards rotation of the thrust axis of the rotors. The interesting characteristic of such systems, as far as VRS is concerned, is that, as the forward speed of the aircraft reduces, the contribution of the lifting surfaces (being proportional to the square of the forward speed of the aircraft) to the deceleration of the vehicle declines and the contribution of the rotors becomes more important.

A simple first-order model can reveal some of the pertinent effects. Application of Newton's second law gives

$$\begin{aligned} m\dot{v}_x &= T\cos\theta - D(v_x, v_z) \\ m\dot{v}_z &= T\sin\theta - L(v_x, v_z) \end{aligned} \tag{6}$$

which can be solved for the rotor thrust $T(t)$ and the thrust vectoring angle $\theta(t)$ once the scheduling law for the aerodynamic lift L and drag D along the trajectory, as well as the trajectory itself, are prescribed.

Figure 6 shows the outcome of an example calculation for such a hybrid configuration where some simplifying assumptions have been made in order to reveal the pertinent effects on the susceptibility of the aircraft to the VRS during its transition from wing-borne to rotor-borne flight, as well as during the subsequent descent to landing. For the purposes of the illustration it is assumed that the angle of attack of the wing is varied to generate all of the lift that is required by the aircraft - until a critical speed is reached, below which the angle of attack of the wing is kept constant and the rotors are used to make up the deficit in lift.

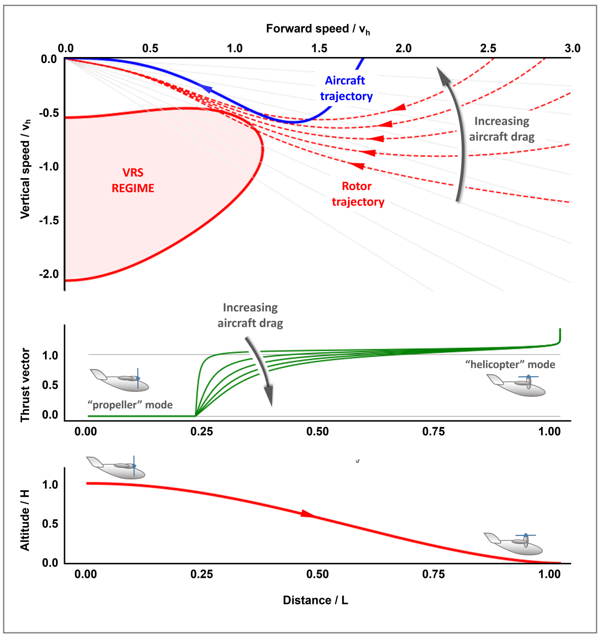


Figure 6
Behaviour near the VRS onset boundary for an eVTOL aircraft with thrust vectoring.

The figure shows the clear effects of the drag of the airframe in changing considerably the susceptibility of the rotors to the onset of the VRS during the transition from wing-borne to rotor-borne flight, as well as during the subsequent deceleration of the aircraft to land. The rearward tilt of the rotors that is required to decelerate the aircraft can, if the conditions are right, be sufficient to create an effective increase in the descent rate of the rotor, as seen in its local reference frame, that is large enough to precipitate the onset of the VRS.

The analysis presented here shows how this effect is exacerbated if the drag of the airframe is too low. This close coupling between the aircraft-like characteristics of the vehicle, in terms of its drag, and the rotorcraft-like characteristics of the vehicle - in terms of its susceptibility to VRS at low forward speeds - potentially poses some difficult choices to the designers of eVTOL aircraft. The traditional aircraft designer's expedient of using spoilers, airbrakes or other drag-producing excrescences to increase the aerodynamic drag of the vehicle during the descent becomes largely ineffective at the speeds over which the vehicle becomes susceptible to VRS. On the other hand, and as alluded to earlier, the obvious alternative solution, which would be to tailor the flight profile to decelerate the aircraft earlier in the descent, may not be entirely consistent with the intended usage for such vehicles.

Another strategy that has quite often been proposed for use on multi-rotor eVTOL aircraft is to employ differential thrust on two or more laterally-spaced rotors to control the roll dynamics of the vehicle. In the second example presented here, such a vehicle is imagined to enter a turn during the last phases of its descent to landing - for instance as might be necessary to negotiate an obstacle or to manoeuvre into an allocated landing spot.

In the example, the thrust on two outboard rotors is varied differentially to instigate a turn to port. The turn is then maintained by keeping the bank angle constant, and finally the turn is completed by cancelling the bank angle using an opposing thrust differential between the outboard rotors. The dynamics of the aircraft can be captured quite effectively by adopting a prescribed variation of bank angle with time, $\phi(t)$, and then realising that, to first order in a coordinated turn,

$$\begin{aligned} v_x &= (v_x)_0 - \epsilon \frac{s}{2} \frac{g}{(v_x)_0} \sin\phi \\ v_z &= (v_z)_0 - \epsilon \frac{s}{2} \dot{\phi} \\ T &= (T)_0 - \epsilon \frac{I}{s} \ddot{\phi} \end{aligned} \qquad (7)$$

where I is the inertia of the aircraft about its roll axis, s is the lateral separation between the rotors that are being used to control the roll dynamics, $\epsilon = 1$ for the port rotor and -1 for the starboard rotor in a turn to port (and vice versa for a turn to starboard) and the notation $()_0$ refers to the nominal value of the respective parameter as measured at the aircraft's centre of gravity.

Figure 7 shows the rather complex trajectory that might be followed with respect to the boundary for VRS onset by the rotors of such a configuration, especially when compared to the nominal trajectory that is followed by the aircraft itself. Initially, the reduction of thrust on the port rotor (and the associated reduction in disc loading) conspires with the roll rate to take the rotor across the VRS boundary. Some recovery is evident as the thrust on the port rotor is increased to maintain the turn, but the yaw rate (and the resultant reduction in in-plane velocity) continues to maintain the port rotor closer to the onset boundary than the starboard rotor throughout the turn. Finally, the thrust increase on the port rotor, together with the roll rate that it induces on the vehicle, acts to move the conditions on the rotor further away from those which might lead to the onset of the VRS. Analogous but opposing effects can be seen on the starboard rotor.

The implication of the analysis for this particular configuration, of course, is that such a manoeuvre, particularly if executed at an inappropriate point in the descent, might lead to a flirtation with the phenomenon known as asymmetric VRS - where the rotors on one side of the aircraft are subject to the

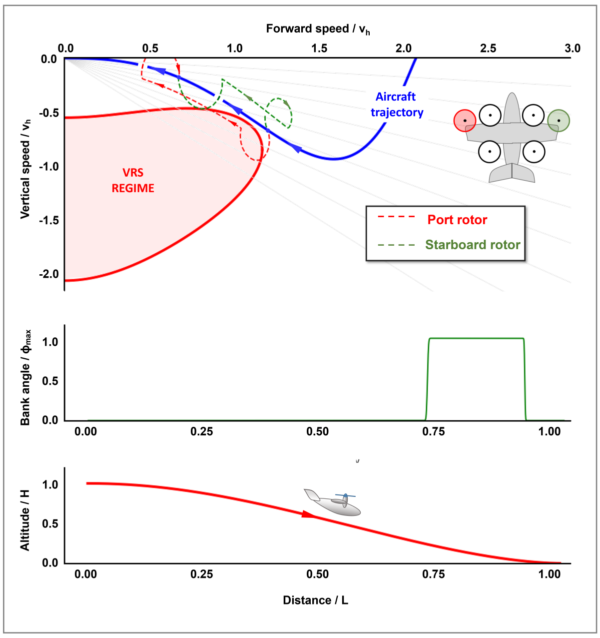


Figure 7
Behaviour near the VRS onset boundary for an eVTOL aircraft in a turn.

VRS while those on the other side are not similarly affected. The associated changes in the control derivatives under such circumstances have proved a particular impediment to the design of effective control systems to counteract the phenomenon even in crewed, semi-conventional systems such as the V-22. It is interesting to speculate how future eVTOL aircraft that have broadly-separated side-by-side rotors might fare if such an effect is inherent in their aero-mechanics, and even more so if the ambition is realised that many of these aircraft be piloted by relatively unskilled personnel - or even that they be fully automated.

Perry's model for the onset of the VRS implicitly assumes the aerodynamics of the system to be quasi-steady. Ahlin and Brown[6] re-formulated the model by paying particular attention to the physical mechanisms within the aerodynamics of the system that lead to the onset of the VRS. Their model, described in the next section of the paper, allows some important additional insights to be gained into the likely behaviour of eVTOL aircraft near to the onset of the VRS under somewhat more unsteady flight conditions than have been analysed so far.

4 GUSTS AND PERTURBATIONS

The wake that is generated by a helicopter rotor is continually subject to small perturbations that act to disrupt its orderly and structured development downstream of the system. These perturbations are simply a result of the various inhomogeneities (for instance, as caused by free turbulence) in the fluid into which the wake develops. The dynamics of the wake in response to these perturbations is known to be unstable, so that perturbations that are initially small grow, given sufficient time, into large-scale disturbances to the structure of the wake. The aerodynamic effect on the rotor of the growth in spatial extent and intensity of these disturbances is kept in check by the ability of the vehicle, given its forward speed and descent rate, to outrun these perturbations (or, in a rotor-fixed frame of reference, for these perturbations to convect away into the wake behind the aircraft).

In Ahlin and Brown's conceptualisation, the rotor enters the VRS when conditions become such that the disturbances in the wake can catch up and engulf the rotor, precipitating an aerodynamic feedback between the rotor and the wake which causes the flow to collapse into the classical, toroidal form that is associated with the phenomenon.

To put this conceptualisation on a firm mathematical footing, it is assumed explicitly that the first disturbance that has sufficient strength to induce the onset of the VRS originates on the rotor itself at some earlier time, and that, during its lifetime, this disturbance has grown in strength whilst simultaneously being convected away from the rotor and into the flow behind the system. The centre of the disturbance will thus have followed the trajectory given by

$$\boldsymbol{x}_d(t, t_*) = \boldsymbol{x}_R(t_*) + \int_{t_*}^{t} \boldsymbol{v}_d(\tau) d\tau \qquad (8)$$

where $\boldsymbol{x}_R(t)$ is the trajectory followed by the rotor, $\boldsymbol{v}_d(t)$ is the velocity with which the disturbance has convected, and t_* is the time of inception of the disturbance at the rotor. The locus of points on which the disturbance has critical strength is given by the set of points $\boldsymbol{x}$ where

$$|\boldsymbol{x} - \boldsymbol{x}_d(t, t_*)| = \int_{t_*}^{t} c_d(\tau) d\tau \qquad (9)$$

and c_d is the rate at which the front on which the disturbance has critical strength has spread out into the flow from the point of inception of the disturbance on the rotor itself.

The system is assumed to enter the VRS when the rotor first comes into contact with a disturbance that has critical strength, or in other words at the earliest time t at which the rotor lies on the locus of points given by Eq. 9. In these terms, the VRS onset boundary is described by the conditions that prevail within the system at the earliest time t at which

$$|\boldsymbol{x}_R(t) - \boldsymbol{x}_d(t, t_*)| = \int_{t_*}^{t} c_d(\tau) d\tau \qquad (10)$$

for all $t_* < t$.

This model for the physics of the VRS is very general and can be used not only to analyse the onset of VRS under quasi-steady conditions, as per Perry's model, but also to lend insight into VRS onset under accelerated flight conditions where the rotor thrust and velocity are changing with time.

A crucial connection to the work presented earlier in this paper is made by realising that Eq. 10 can be made to reduce to Perry's model if the velocity and thrust of the rotor are assumed to be constant (or at least to be quasi-steady) and we define

$$\dot{\boldsymbol{x}}_R = (v_x, v_y, v_z), \quad (11)$$
$$\dot{\boldsymbol{x}}_d = ((1-k)v_x, 0, -v_i) \quad (12)$$

and

$$c_d = \bar{\mu}_W \sqrt{W_\rho}. \quad (13)$$

This inherent consistency between the two formalisms allows the more sophisticated phenomenological basis for Eq. 10 to be used to generate versions of Perry's simpler model that can be used under more complex circumstances than those for which the original model was developed.

So, for example, Eq. 10 allows a 'VRS margin' $Q(t)$ to be defined (with units of distance) such that

$$Q(t) = \min_{t_*} \left[|\boldsymbol{x}_R(t) - \boldsymbol{x}_d(t, t_*)| - \int_{t_*}^{t} c_d(\tau) d\tau \right] \quad (14)$$

or, in other words, as the shortest distance, at any particular instant, between the rotor and those disturbances within the rotor wake that have critical strength. It is quite simple to show that the analogous margin $\bar{q}$ (with 'units' of non-dimensional velocity) in Perry's model obeys

$$(k\bar{\mu}_x)^2 + (\bar{\mu}_z + \bar{\lambda}_i)^2 = (\bar{\mu}_W + \bar{q})^2 \quad (15)$$

The utility of this margin is that it is zero when the conditions within the system place it on the VRS onset boundary, and negative with the system within the VRS. The margin becomes increasingly more positive as the flight conditions of the system move it further outward on the $(\bar{\mu}_x, \bar{\mu}_z)$ diagram, away from the boundary for VRS onset.

A practical application of Eq. 15 is to use it to estimate the susceptibility of the aircraft at any point in its flight envelope to the onset of the VRS as a result of the effects of natural air turbulence. The pilots of light helicopters, for instance, are well aware of the dangers posed by gusts from behind or below during the most critical parts of their descent to landing, and it is instructive to contrast the likely behaviour of eVTOL aircraft on a similar basis. As remarked earlier, susceptibility to gusts could become an especially pertinent consideration for these new vehicles, particularly given the complex, unsteady air currents to which some of them are likely to be exposed when operating in the urban environments for which they have been designed.

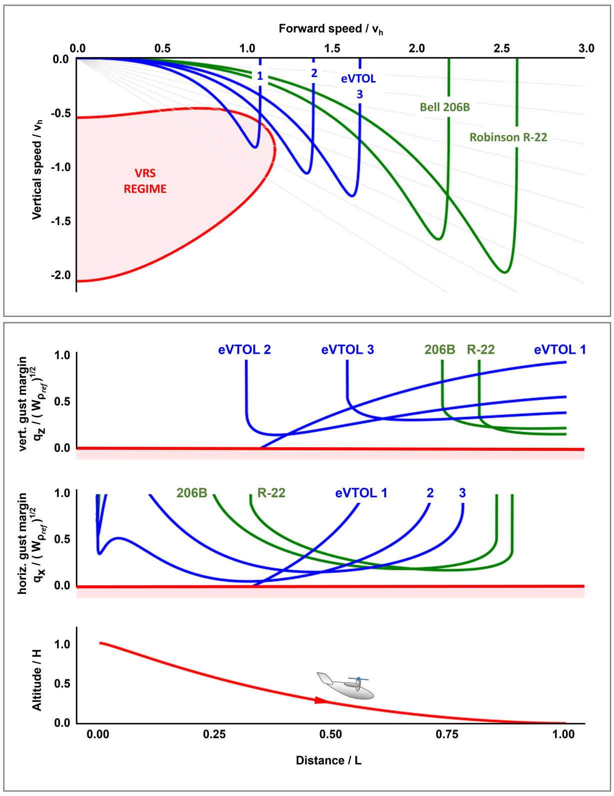


Figure 8
Gust margins for eVTOL aircraft and conventional helicopters during descent.

If the 'margin of susceptibility' to gusts is defined simply as the speed of the strongest gust that can be encountered without the aircraft being precipitated into VRS conditions, then Eq. 15 can be transformed into an expression that gives the (non-dimensional) margin of susceptibility to horizontal tailwind-like gusts as

$$\bar{q}_x = k\bar{\mu}_x - \sqrt{\bar{\mu}_W^2 - (\bar{\mu}_z + \bar{\lambda}_i)^2} \quad (16)$$

and similarly the margin of susceptibility to vertical updraught-type gusts as

$$\bar{q}_z = (\bar{\mu}_z + \bar{\lambda}_i) - \sqrt{\bar{\mu}_W^2 - (k\bar{\mu}_x)^2} \quad (17)$$

The dimensional equivalents $q_x = \bar{q}_x\sqrt{W_\rho}$ and $q_z = \bar{q}_z\sqrt{W_\rho}$ to these margins then follow.

As an example of the use of Eq. 15 in practice, Fig. 8 compares the gust margins for the same range of eVTOL aircraft and light conventional helicopters as presented in previous figures, when all follow the same descent trajectory. Note that the gust margins in this figure are presented in essentially dimensional form (albeit scaled by a representative reference disc

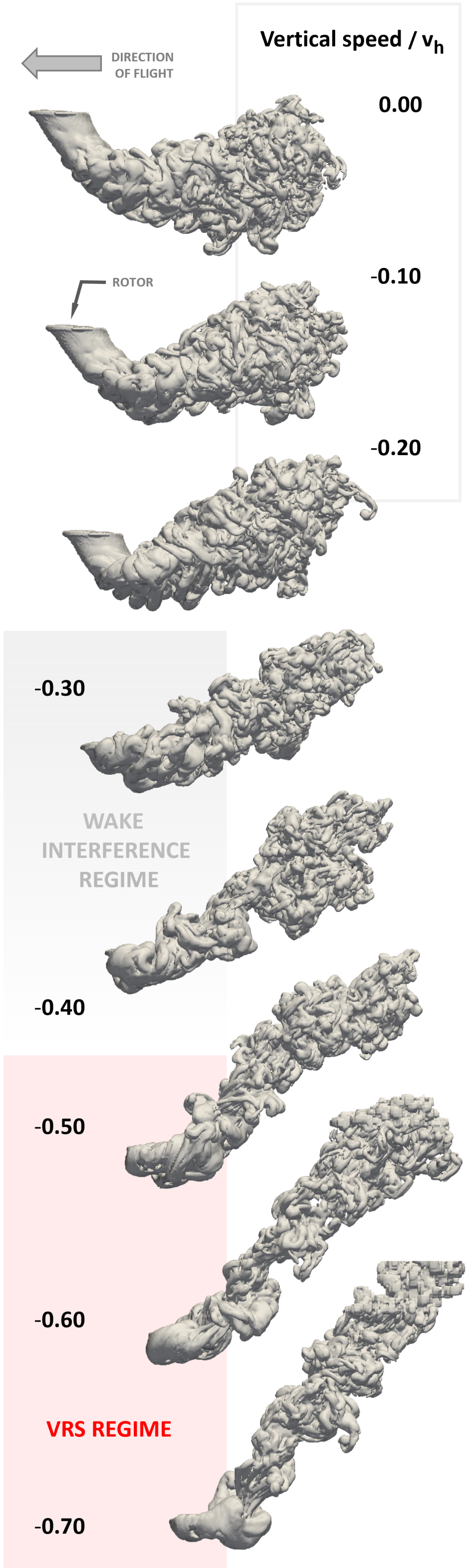


Figure 9
The wake that is produced by an isolated rotor at various rates of descent, as predicted by SophIA-C.

loading so as to normalise the axes appropriately) to be able to compare the characteristics of the various different types of vehicle on an even basis.

For the trajectory as shown, a clear margin in terms of both horizontal and vertical gusts that the vehicles can encounter without being precipitated into VRS conditions is evident for both of the two light helicopters. Consistently with operational experience, however, both vehicles are predicted by the analysis to be exposed to a prolonged period during the descent during which they are particularly susceptible to horizontal, tailwind-like gusts.

Along this particular descent trajectory, 'eVTOL 3' has similar gust margins to the light helicopters, although the constraints imposed on its dynamics by the possibility of VRS onset begin much earlier on during its descent. In contrast, the behaviour of 'eVTOL 2' displays a distinct choke point in terms of its gust response: the vehicle becomes very sensitive to both horizontal and to vertical gusts as the descent trajectory passes the 'knee' of the VRS onset boundary for this particular aircraft. According to the model, 'eVTOL 1' would not be able to negotiate this particular descent trajectory without falling foul of the VRS - it has no gust margin at all during the earliest portions of the descent.

The analysis presented here shows how the often-quoted advantage of eVTOL aircraft's high disc loading in reducing the sensitivity of the aircraft to gusts (at least in terms of the associated reduction in the resultant accelerations of the aircraft) might turn into a distinct disadvantage if the trajectory of the aircraft is not optimised to take into account the proximity of the VRS onset boundary during the most susceptible phases of its descent. In that vein, it is worth pointing out that a particularly useful feature of the gust margin when defined in the form of Eqs. 16 and 17 is that it can be incorporated readily as a constraint within the process of optimising the trajectories of such vehicles in order to minimise the likelihood of their being susceptible to the onset of VRS during their descent to landing.

In the next and final technical section of this paper, we introduce a methodology that can lend insight into one of the most interesting and difficult-to-address problems in the design of eVTOL aircraft by accounting for some of the effects of rotor aerodynamic interaction on the susceptibility of multi-rotor systems to the onset of the VRS.

We use the accelerated VRS model presented in this section of the paper to form the basis of our interpretation of the data and to make some observations regarding the likely effects of aerodynamic interactions on the susceptibility of many current eVTOL aircraft to the VRS - particularly given the prevalence of certain common design features that might exacerbate their behaviour in this dangerous flight regime.

5 AERODYNAMIC INTERACTIONS

The developmental history of conventional rotorcraft is full of examples where aerodynamic interactions that were unforeseen during the design of the aircraft have ended up emerging as the principal cause of safety or performance issues once the aircraft has gone forward to flight testing[8]. In many cases, the process of rectifying these problems has led to significant delays and cost overruns in getting the aircraft ready for introduction into service. In many of these instances, a contributory factor was that the designers had under-appreciated how the aerodynamic interactions within the system would come to dominate the flight physics of the aircraft, particularly if there was a strong incentive to package the rotors and lifting surfaces into as compact a configuration as possible.

Based on this experience, it is hard to see how the drive towards ultra-compactness in the modern generation of eVTOL aircraft, often resulting in very compact configurations with multiple rotors and lifting surfaces operating in close proximity, might not lead to similar issues to those already experienced within the helicopter community.

Indeed, on extrapolating from current knowledge, particularly from within the tilt-rotor community, it is quite concerning to note that the design of a fair proportion of current eVTOL aircraft is converging on a configuration that embodies a number of elements, such as closely-spaced side-by-side rotors, and rotors operating in tandem, that have been implicated in previous design studies (or even, in some cases, in flight trials) as having been responsible for degrading the handling qualities of the aircraft when operating near to the onset of the VRS, or even in precipitating an overly abrupt or early onset of VRS conditions within the system. Given the early stage of development of most eVTOL concepts, it would thus seem opportune to attract designers' attention to the possible pitfalls in some of their configurational choices as far as the VRS is concerned. In this section of the paper we thus explore a very small and somewhat abstracted sample of such effects in in order to expose some of the basic behaviour that might be expected.

The complexity of the aerodynamics that are a feature of such interactions makes them particularly difficult to characterise using simple analytical models of the type that has been exploited extensively up to this point in the paper. These simpler models continue to have great value though in helping to interpret and condense the output of more comprehensive modelling approaches. Indeed, the approach followed here will be to use the conceptualisation of the fluid dynamics of the VRS that was introduced in the previous section of the paper to frame and interpret the results of a small set of numerical simulations that have been conducted using techniques that are able to resolve the detailed structure and dynamics of the vehicle's wake as it evolves downstream of the aircraft.

The numerical simulations are performed using Sophrodyne Aerospace's new SophIA-C rotorcraft simulation environment. This approach calculates the aerodynamics of the wake using a modern implementation of the Vorticity Transport Model (or VTM). The VTM[9] is a computational approach to rotor wake modelling that is well-established within the rotorcraft community through having been tested on applications ranging from the characterisation of rotor-induced brownout[10] through to extensive application in the analysis of aerodynamic interactions in complex helicopter configurations[11], and, indeed, to the study of the onset of the VRS itself[12].

The VTM is based on a numerical solution of the Navier-Stokes equations, cast into velocity-vorticity form (rather than the velocity-pressure form of most traditional rotorcraft CFD techniques) on a computational grid surrounding the vehicle. The principal advantage of this approach over more conventional CFD techniques for modelling the dynamics of the wake is that the vortical structures and subsequent wake dynamics that are responsible for the onset of the VRS are preserved intact for the many rotor revolutions that are required for the instabilities within the rotor wake to propagate fully through the system.

Its unique formulation makes the VTM particularly well suited to capturing the effects of the interaction between the rotors and other parts of the airframe that are a feature of the aerodynamics of the compact, multi-rotor configurations that are characteristic of current eVTOL designs. In contrast, standard CFD techniques tend to require inordinate computational resources and very fine resolution of the flow if these same vortical structures are not to be diffused and dissipated into the flow a short distance downwind of the rotor from where they originate[3].

To establish a baseline against which later results can be compared, Fig. 9 shows the results of a SophIA-C simulation of the changes in wake structure that occur when an isolated rotor is accelerated into descent. In the simulation, the rotor is kept at a horizontal velocity $\bar{\mu}_x = 0.5$ throughout (in other

[3] Indeed, when using these inherently dissipative methodologies there is a serious risk that many of the aerodynamic interactions within the system will be characterised as weak, short-range and benign even when they are not in fact so. There is thus a significant risk of under-appreciating the likely dangers that are posed to the safety of the system when such tools are used without due regard to their inherent limitations.

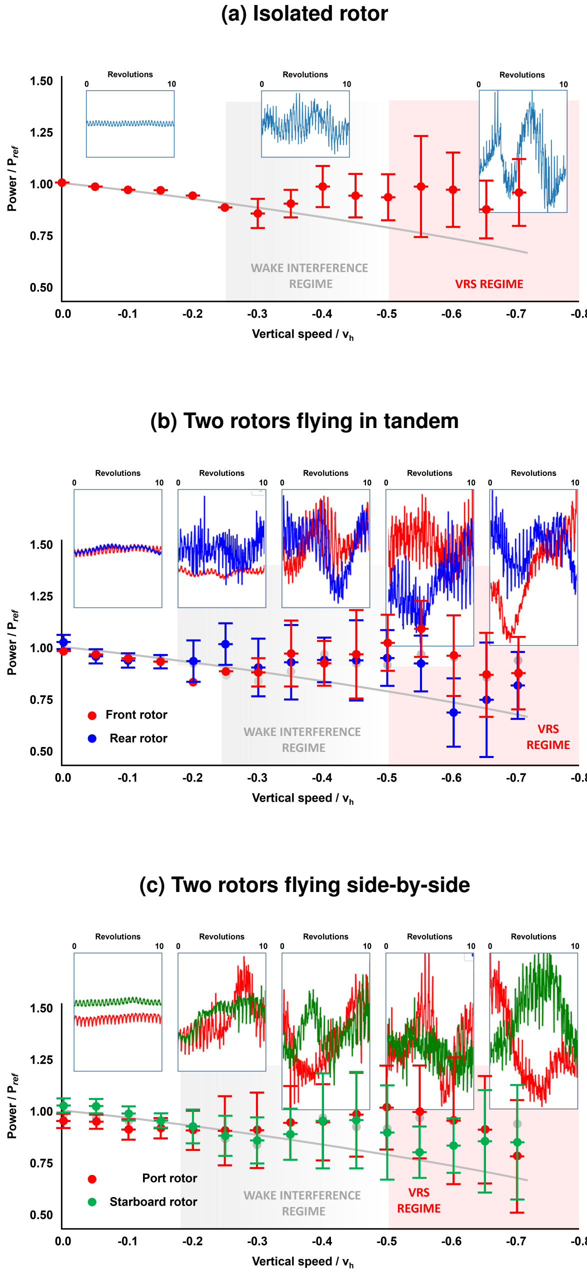


Figure 10

The power that is required by various rotor systems during descent into the VRS when operating at constant thrust, as predicted by SophIA-C.

words, on comparing to Fig. 2, at a forward speed at which the rotor will enter the VRS if its descent rate is increased much beyond where $\bar{\mu}_z = -0.5$). The upper figure has the rotor in level flight; in each subsequent figure the descent rate of the system is increased in equal increments so that the bottom figure has the rotor descending with $\bar{\mu}_z = -0.7$. The rotor is controlled via its collective pitch to keep the thrust constant throughout the descent. Note that in the eVTOL domain it is far more common to use rotational speed rather than blade feathering to control the thrust; the intent behind the selection of the control strategy for the simulations presented here, however, is to avoid the time-lags that are introduced into the system by the rotor inertia, when using rotor speed control, from obscuring the basic, underlying physics of the VRS. A concession to typical eVTOL practice, where stiff hubs are the norm, is made however by modelling the blades to be rigid - in other words no flapping or lagging about the blade roots is allowed to take place in the simulation.

The sequence of figures shows very clearly the development of the wake dowstream of the rotor[4] and how the portion of the wake that is disrupted by the inherent instability within its vortical structure encroaches ever more closely on the rotor as the descent rate of the system is increased. In fact, two rather abrupt changes in the overall behaviour of the wake are evident. At a descent rate where $\bar{\mu}_z \approx -0.25$, the highly disturbed portion of the wake starts to impinge directly on the rotor, and the wake responds by rolling up to form a horseshoe-shaped structure, the leading edge of which tends to bob up and down just below the rotor disc itself. The flow in this 'wake interference' regime is characterised by strong interaction between the rotor and the coalesced products of the destabilisation of its wake, but the interaction is insufficiently powerful to precipitate the rotor into the VRS proper. Eventually, at a non-dimensional descent rate where $\bar{\mu}_z \approx -0.5$, the interaction of these disturbances with the rotor precipitates a distinct topological change in the structure of the wake during which the flow reconfigures into the toroidal form that is classically associated with the onset of the Vortex Ring State. The dynamics of the vorticity within this toroidal structure are described comprehensively elsewhere[13], but the end-product of the intense re-orientation and stretching within the torus of the vorticity that is produced by the rotor blades can be seen in the highly-disordered but concentrated stream of vorticity that is ejected upwards from the centre of the rotor.

These changes in wake structure are associated with some very distinct changes in the loads that are produced on the rotor system itself. Figure 10a shows the power that is consumed by the rotor as a function of its rate of descent. For rates of descent that are too small to precipitate the VRS, the power follows a very similar trend to that predicted by simple momentum theory, and the fluctuations about the mean are relatively small. Within the wake impingement regime, the fluctuations in the power increase quite substantially, and are characterised by high-frequency content that is symptomatic of the

[4]The structure of the wake is visualised by plotting one of the surfaces within the flow on which the vorticity has constant magnitude. A relatively low value of vorticity is chosen to reveal the gross characteristics of the wake - smaller-scale, more intense structures, such as the individual tip vortices from each of the blades, are hidden within the surface that has been plotted

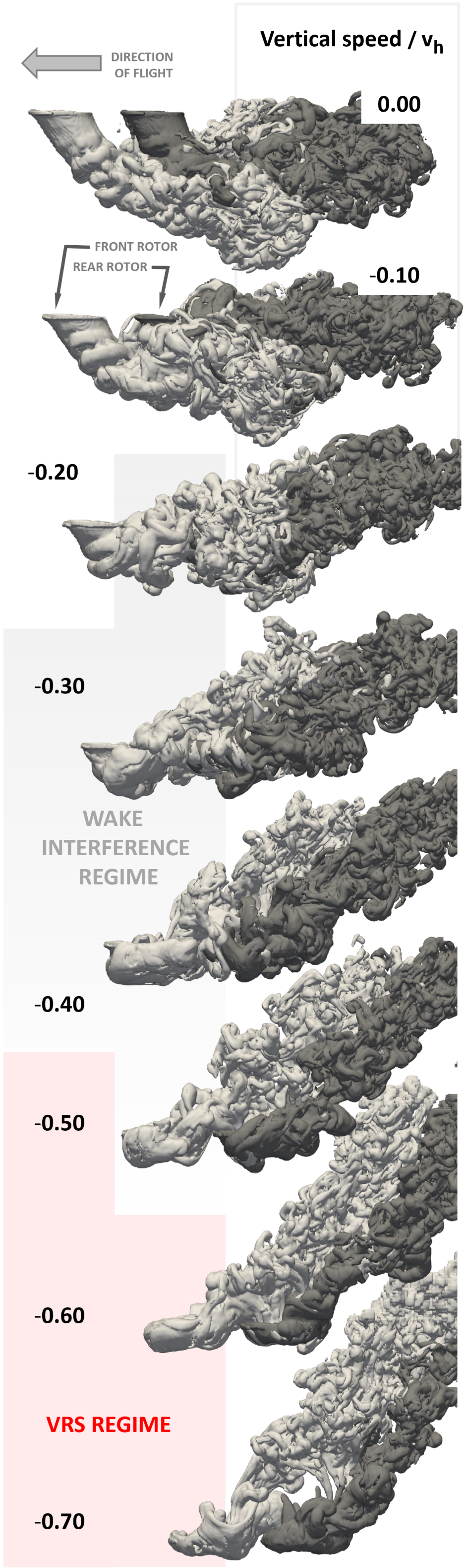


Figure 11
The wake that is produced by a pair of rotors flying in tandem at various rates of descent, as predicted by SophIA-C.

increasing importance of aerodynamic interactions between the blades and the individual tip vortices that are produced by the rotor. A lower-frequency component to the fluctuations in loading is attributable to the unsteadiness referred to earlier in the overall structure of the wake with the rotor in the wake impingement regime. With the rotor within the VRS, the character of the fluctuations changes markedly again - the BVI-related excitation of the system is superimposed on a succession of large and persistent excursions in the loading on the rotor. Closer examination of the output of the simulations shows these large fluctuations to be a result of gross changes in the shape and location of the regions of most intense vorticity within the toroidal structure of the collapsed rotor wake, accompanied by the occasional collapse of the torus and the associated shedding, referred to earlier, of large clumps of disordered vorticity into the flow downstream of the system. Notable too is the overall higher power consumption of the rotor that is required to keep its thrust constant post the onset of the wake interference regime than might have been expected from simpler analysis.

As a first example showing the effects of aircraft configuration on the VRS-related characteristics of the system, Fig. 11 captures the essence of the interactional aerodynamics of a particularly common configurational element of modern eVTOL design - namely where pairs of rotors are used in tandem to even out the thrust-induced pitching moments about the centre of gravity when the vehicle is operated in ‘helicopter’ mode. The figure shows the evolution of the wakes[5] of a closely-spaced pair of rotors when they are flown in tandem under the same conditions as were used to generate Fig. 9. The horizontal distance between the rotor axes (set to three rotor radii in the simulation presented here) is broadly representative of current practice in multi-rotor eVTOL aircraft design, and exposes particularly well the physics of the interaction. Both rotors are controlled throughout the course of the simulation, using their collective pitch to maintain their thrust at the same setting as for the isolated rotor in the previous example.

Figure 11 (and Fig. 10b for the power) shows the front rotor of the pair in tandem to follow much the same evolution, at least superficially, as the isolated rotor as the descent rate of the system is increased - both in terms of the development of its wake and in terms of the loads that are produced on the system. Compared to the front rotor (which would appear to experience only the longer-range, weaker, induced-velocity effects from the rotor behind it), it might be expected that the rear rotor would experience much stronger and more direct interactional effects through being immersed in the wake of the rotor in front of it over much of the range of simulated descent rates.

[5] The vorticity created separately by each of the rotors is rendered in a different shade of grey in order to expose some of the finer details of the interaction.

This is true up to a point.

At the lowest descent rates, the wakes of both rotors are swept sharply downwards into the flow below the system, and the interactional effects are relatively mild. The rear rotor experiences a small decrease in its power output and a very mild level of unsteadiness in its loading as a result of its interaction with the front rotor, but the front rotor is left largely unperturbed by the interaction with the rotor to its rear. As the descent rate of the system is increased and the de-stabilised portion of the wake of the front rotor begins to move forwards and upwards closer to the disc plane of the rear rotor, the increasing strength of the interaction between the rear rotor and the coalesced vortical disturbances within this wake leads to the onset of marked fluctuations in the loading on the rear rotor. These fluctuations have relatively large amplitude and low frequency. As the descent rate is increased further, a very important development takes place - this interaction begins to feed back into the dynamics of the wake of the front rotor itself, causing the amplitude of the fluctuations in its loading to increase well above those that characterise its behaviour at the same descent rates when operating in isolation.

Even once the trajectory of the system is sufficiently steep for most of the wake of the front rotor to be convected above the plane of the rear rotor, filaments of intense vorticity are intermittently drawn down from the de-stabilised portion of the wake of the front rotor and ingested through the disc of the rear rotor. This interaction leads to particularly unsteady aerodynamic loads on the system, with amplitudes that are only marginally smaller than those that are encountered by the rotors when in the VRS proper. This constant perturbation of the aerodynamics of the rear rotor through interaction with the most unsteady portions of the wake of the front rotor only begins to abate at the highest simulated descent rates, once the front rotor has entered the VRS proper. By that stage, conditions are such that the rear rotor has been precipitated into its own VRS, and thus the system finds itself in a state where the wake of both rotors has collapsed, and both rotors are subject to large, aperiodic excursions in loading. The fact that the fluctuations in loading on the front and rear rotors are only weakly synchronous, if at all, is very likely to add an extra element of unsteadiness to the forcing of a system that is subject to this form of multi-rotor VRS.

This simulation is particularly instructive, revealing as it does how a closely-coupled, tandem arrangement of rotors might lead to a regime of highly unsteady, large-amplitude VRS-like loads on the system which extends to descent rates that are well below those at which either rotor within the system might ostensibly

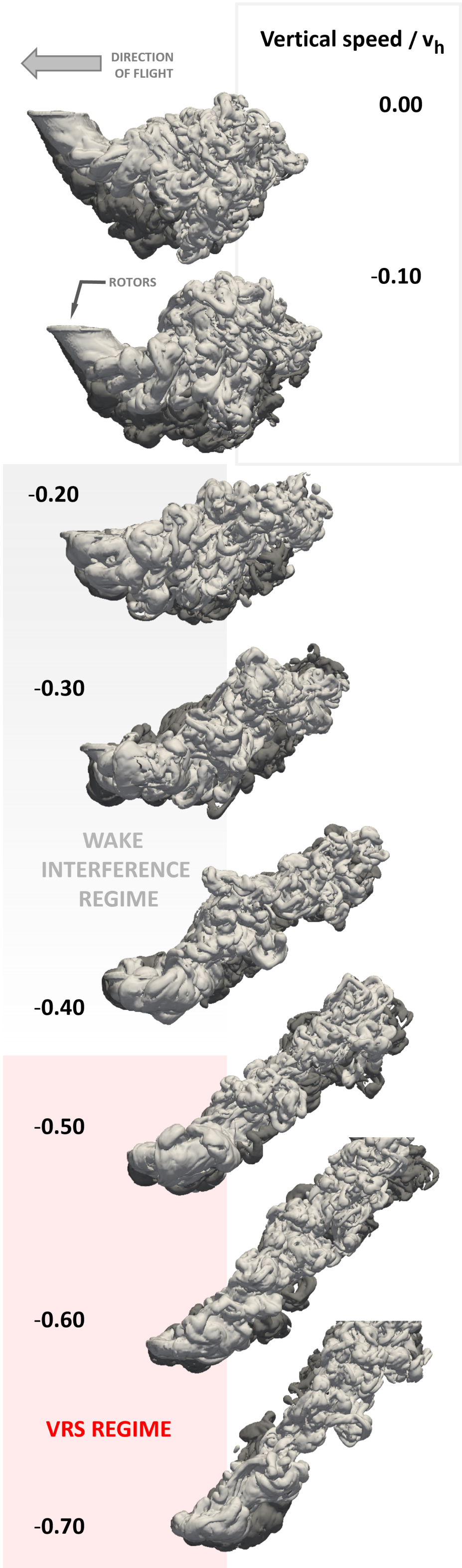


Figure 12
The wake that is produced by a pair of rotors when flying side-by-side at various rates of descent, as predicted by SophIA-C.

enter the VRS proper. The underlying mechanism is the immersion of the trailing rotor in the wake of the leading rotor as it de-stabilises and collapses into the VRS, and the feedback of the perturbations to the

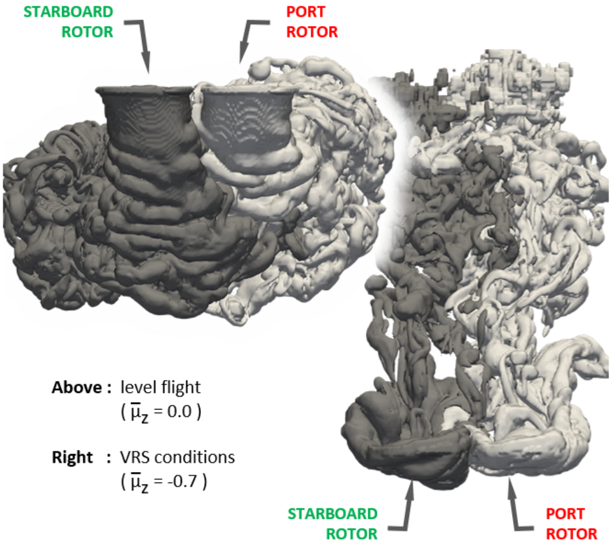


Figure 13
View from forward of the wakes that are produced by two rotors when flying side-by-side, as predicted by SophIA-C.

loads on the rear rotor into the aerodynamics of the leading rotor to create a self-sustaining fluctuation in the loads on the entire system.

As a second example, Fig. 12 (and Fig. 10c for the power) captures the principal features of the interactional aerodynamics of yet another commonly-encountered configurational element within the design of modern eVTOL aircraft, namely where an array of rotors is mounted closely side-by-side, as for instance as an embodiment of the Distributed Electric Propulsion concept. The lateral separation between the rotor axes in this case was set to be equal to 2.25 times the rotor radius in order to emulate the characteristics of some of the more compact current eVTOL aircraft configurations. Both rotors in the simulation rotate in the same sense (clockwise when viewed from above), and both are controlled as previously described in order to produce the same thrust throughout the simulated descent.

The destruction of any obvious lateral symmetry within the system when both rotors rotate in the same sense has very interesting consequences for the physics of their interaction. A primary contributor to the aerodynamics of the system is the inability of the blades on either rotor to flap or to be cyclically feathered. Although this is a particularly common design simplification that is embodied in many eVTOL aircraft, its effect here is to create a strong lateral asymmetry in the loading on each rotor disc which manifests as a lateral asymmetry in the strength of their wakes - the wake is strengthened on the port side of each rotor, and weakened on the starboard side, for the sense of rotation simulated here. Bear in mind that the flow induced by a rotor is generally upwards everywhere further outboard than the rotor tips themselves[6]. At the lowest descent rates the effect of this asymmetry is thus to cause the wake of the port rotor to lift above that of the starboard rotor (see Fig. 13), effectively causing this rotor to behave as though the system is operating at higher descent rate than it actually is. The effect of the interference from the neighbouring rotor is thus to cause the regime of wake interference for the port rotor to begin at appreciably lower descent rates than it would have with the rotor in isolation. The perturbations to the wake of the port rotor strongly influence the development of the wake of the starboard rotor, and, as their descent rate is increased, both rotors enter a protracted regime of very unsteady loading prior to their wakes eventually succumbing to their inherent instability and reconfiguring into the toroidal form that is characteristic of the VRS (again, see Fig. 13).

This simulation thus reveals how relatively long-range aerodynamic effects can couple the dynamics of several of the rotors and exacerbate the inherent instability of the wake that is responsible for the eventual onset of VRS conditions. In this case again, the aerodynamic interaction between the rotors leads to a regime of highly unsteady, large-amplitude VRS-like loads on the system which, as in the case of the rotors flying in tandem, but for subtly different reasons, extends to descent rates that are well below those at which either rotor within the system might ostensibly enter the VRS proper.

Space unfortunately permits only a few sample calculations to be presented here, but those cases that have been presented have been selected to be indicative of the broad range of interactional effects that might need to be examined in detail during the design of highly compact multi-rotor aircraft, such as embodied by the present crop of eVTOL machines, in order to ensure their safety as far as the onset of the VRS is concerned. Obviously the exact form and magnitude of the effects will be dependent on the parameters of the vehicle in question, but it is hoped that the information provided here will be of use to designers, particularly in motivating them to consider the broader implications of certain configurational decisions that might have been made originally on more straightforward grounds.

6 CONCLUSION

Worldwide, companies large and small are competing to produce small, lightweight multi-rotor, electric VTOL aircraft to satisfy a perceived market need for rapid, congestion-free intra-urban transport. These vehicles promise to usher in a new world of environmentally-friendly convenience, but in order to

[6] The analogy with the flow created by a wing with very low aspect ratio might be useful in visualising the relevant aerodynamics.

do so, especially within the urban environment, the burden falls upon their designers to fully and properly assess their safety under all possible operational conditions. A particularly hazardous operating condition, called the Vortex Ring State, can occur during the descending flight of conventional helicopters. There is a history of mishap involving this condition, during which the rotors lose thrust and, even if sufficient altitude exists in which to recover the aircraft, the piloting response that is required to alleviate or escape the condition can be counterintuitive.

When extrapolating helicopter experience to this new class of eVTOL aircraft, and particularly in incorporating experience with aircraft that lie somewhat off the established design trend, such as the V-22 Osprey, it quickly becomes apparent that some of the features of the design of these new aircraft might act to exacerbate the possibility of VRS- related flight dynamic issues during their descent and landing, especially when compared to the behaviour of more conventionally-configured rotorcraft. Their high disc loading compared to conventional helicopters, as well as more subtle features of their design, such as their propeller-like blade twist and the details of their blade tip shape, may conspire together with the compact, multi-rotor, distributed-propulsion type configuration that is characteristic of many of these modern designs, to produce VRS-related behaviour during descent that, although similar in its physical origins to that on more conventional rotorcraft, is very different in character.

This paper has provided results from a hierarchy of simulations of the coupled aerodynamics and flight mechanics of vehicles with various representative features of this new class of aircraft. The aim has been to provide insight into the likely characteristics of the Vortex Ring State to which these vehicles might be exposed in descending flight, as well as to make a start in identifying and constructing the rigorous engineering means by which any problems that are identified might be analysed and addressed.

The author has been particularly careful to represent the consequences of design or operation on the VRS-related characteristics of these vehicles in as general and as qualitative a fashion as possible, and the danger of course is that this generality will be construed by some readers as an attempt to pick on eVTOL aircraft as a uniformly-flawed class of vehicles as a whole. Whilst it is true that certain of their characteristic design features and operational strategies may inherently exacerbate the onset of the VRS, not all eVTOL aircraft embody the entire range of features that are catalogued in this paper, and thus it would be false to believe that all eVTOL aircraft are unequivocally more susceptible to the onset of the VRS than conventional helicopters under all possible conditions.

Indeed, the devil is most certainly in the detail, and it is true to say that many of the pitfalls described here can be avoided by properly-informed and conscious application of certain rather basic design principles. Undoubtedly this paper will not be the last word on the subject, but it is hoped that some of the issues raised herein will motivate and challenge the designers of eVTOL aircraft to take into account, at a very early stage in their thinking, the potential for the rotors of their particular creation to enter this potentially catastrophic aerodynamic condition. The hope is thus that this work will contribute positively to the creation of a new class of flying machines that safely and reliably achieves the loftiest ambitions of its most enthusiastic proponents for a new, environmentally-friendly, and easily-accessible means of urban transport.

References

[1] STEWART, W., "Helicopter Behaviour in the Vortex-Ring Conditions," Aeronautical Research Committee R&M No. 3117, November 1951.

[2] BRAND, A., KISOR, R., BLYTH, R., MASON, D., and HOST, C., "V-22 High Rate of Descent (HROD) Test Procedures and Long Record Analysis," *60th American Helicopter Society Annual National Forum*, Baltimore MD, USA, 7-10 June 2004.

[3] BROWN, R.E., NEWMAN, S.J., PERRY, F.J., AHLIN, G.A., MODHA, A., and KHELIFA, M., "Revisiting the Effects of Blade Geometry on Rotor Behaviour in Descending Flight," *30th European Rotorcraft Forum*, Marseilles, France, 14-16 September 2004.

[4] JOHNSON, W., "Model for Vortex Ring State Influence on Rotorcraft Flight Dynamics," *AHS 4th Decennial Specialist's Conference on Aeromechanics*, San Francisco CA, USA, 21-23 January 2004.

[5] BROWN, R.E., NEWMAN, S.J., PERRY, F.J., ORCHARD, M., MODHA, A., and LEWIS, S., "Predicting the Onset of Wake Breakdown for Rotors in Descending Flight," *Journal of the American Helicopter Society*, Vol.48, No.1, 2003, pp. 28-38.

[6] AHLIN, G.A., and BROWN, R.E., "Predicting the Onset of the Vortex Ring State under Accelerated Flight Conditions," *61st American Helicopter Society Annual Forum*, Grapevine TX, USA, 1-3 June 2005.

[7] AHLIN, G.A., and BROWN, R.E., "Wake Structure and Kinematics in the Vortex Ring State," *Journal of the American Helicopter Society*, Vol.54, No.3, 2009, pp. 032003-1 - 032003-18.

[8] FLETCHER, T.M., and BROWN, R.E., "Main Rotor - Tail Rotor Interaction and its Implications for Helicopter Directional Control," *Journal of the American Helicopter Society*, Vol.53, No.2, 2008, pp. 125-138.

[9] BROWN, R.E., and LINE, A.J., "Efficient High-Resolution Wake Modeling Using the Vorticity Transport Equation," *AIAA Journal*, Vol.43, No.7, 2005, pp. 1434-1443.

[10] PHILLIPS, C., and BROWN, R.E., "Eulerian Simulation of the Fluid Dynamics of Helicopter Brownout," *AIAA Journal of Aircraft*, Vol.46, No.4, 2009, pp. 1416-1429.

[11] KIM, H.W., KENYON, A.R., DURAISAMY, K., and BROWN, R.E., "Interactional Aerodynamics and Acoustics of a Hingeless Coaxial Helicopter with an Auxiliary Propeller in Forward Flight," *Aeronautical Journal of the Royal Aeronautical Society*, Vol.113, No.1140, 2009, pp. 65-78.

[12] BROWN, R.E., LINE, A.J., and AHLIN, G.A., "Fuselage and Tail-Rotor Interference Effects on Helicopter Wake Development in Descending Flight," *60th American Helicopter Society Annual National Forum*, Baltimore MD, USA, 7-10 June 2004.

[13] AHLIN, G.A. and BROWN, R.E., "The Vortex Dynamics of the Rotor Vortex Ring Phenomenon.," *63rd American Helicopter Society Annual Forum*, Virginia Beach VA, USA, 1-3 May 2007.